UR-1612
ER/40685/950
SEPTEMBER 2000

RECENT RESULTS ON BFKL PHYSICS

LYNNE H. ORR
Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627-0171, USA
E-mail: orr@pas.rochester.edu

W.J. STIRLING
Departments of Physics and Mathematical Sciences, University of Durham
Durham DH1 3LE, England
E-mail: W.J.Stirling@durham.ac.uk

Virtual photon scattering in e^+e^- collisions can result in events with the electron-positron pair produced at large rapidity separation in association with hadrons. The BFKL equation resums large logarithms that dominate the cross section for this process. After a brief overview of analytic BFKL resummation and its experimental status, we report on a Monte Carlo method for solving the BFKL equation that allows kinematic constraints to be taken into account. We discuss results for e^+e^- collisions using both fixed-order QCD and the BFKL approach. We conclude with some brief comments on the status of NLL calculations.

Presented at XXXth International Conference on High Energy Physics, Osaka, Japan, July 27 – August 2, 2000

1 Introduction

Many processes in QCD can be described by a fixed order expansion in the strong coupling constant α_S. In some kinematic regimes, however, each power of α_S gets multiplied by a large logarithm (of some ratio of relevant scales), and fixed-order calculations must give way to leading-log calculations in which such terms are resummed. The BFKL equation [1] resums these large logarithms when they arise from multiple (real and virtual) gluon emissions. In the BFKL regime, the transverse momenta of the contributing gluons are comparable but they are strongly ordered in rapidity.

The BFKL equation can be solved analytically, and its solutions usually result in (parton-level) cross sections that increase as the power λ, where $\lambda = 4C_A \ln 2\, \alpha_s/\pi \approx 0.5$.[a] For example, in dijet production at large rapidity separation Δ in hadron colliders,[2] BFKL predicts for the parton-level cross section $\hat\sigma\ e^{\lambda\Delta}$. In virtual photon scattering, for example in e^+e^- collisions, where the electron-positron pair at emerge with a large rapidity separation and hadronic activity in between,[3] BFKL predicts $\sigma_{\gamma^*\gamma^*}\ (W^2/Q^2)^\lambda$. W^2 is the invariant mass of the hadronic system (equivalently, the photon-photon center-of-mass energy) and Q^2 is the invariant mass of either photon.

2 Experimental Status and Improved Predictions

The experimental status of BFKL is ambiguous at best, with existing results being far

[a] λ is also known as $\alpha_P - 1$.

from definitive. The data tend to lie between the predictions of fixed-order QCD and analytic solutions to the BFKL equation. This happens, for example, for the azimuthal decorrelation in dijet production at the Fermilab Tevatron[4] and for the virtual photon cross section at LEP.[5] Similar results are found in *ep* collissions at HERA.[b]

It is not so surprising that analytic BFKL predicts stronger effects than seen in data. Analytic BFKL solutions implicitly contain sums over arbitrary numbers of gluons with arbitrary energies, but the kinematics are leading-order only. As a result there is no kinematic cost to emit gluons, and energy and momentum are not conserved, and BFKL effects are artifically enhanced.

This situation can be remedied by a Monte Carlo implementation of solutions to the BFKL equation.[7,8] In such an implementation the BFKL equation is solved by iteration, making the sum over gluons explicit. Then kinematic constraints can be implemented directly, and conservation of energy and momentum is restored. This tends to lead to a suppression of BFKL-type effects. The Monte Carlo approach has been applied to dijet production at hadron colliders,[7,8,9] leading to better (though still not perfect) agreement with the dijet azimuthal decorrelation data.[7] Applications to forward jet production at HERA and to virtual photon scattering in e^+e^- collisions are underway; an update on the latter appears in the next section.

3 $\gamma^*\gamma^*$ Scattering: A Closer Look

BFKL effects can arise in e^+e^- collisions via the scattering of virtual photons emitted from the initial e^+ and e^-. The scattered electron and positron appear in the forward and backward regions ("double-tagged"

[b]One exception is the ratio of the dijet production cross sections at center of mass energies 630 GeV and 1800 GeV at the Tevatron, where the measured ratio lies above *all* predictions.[6]

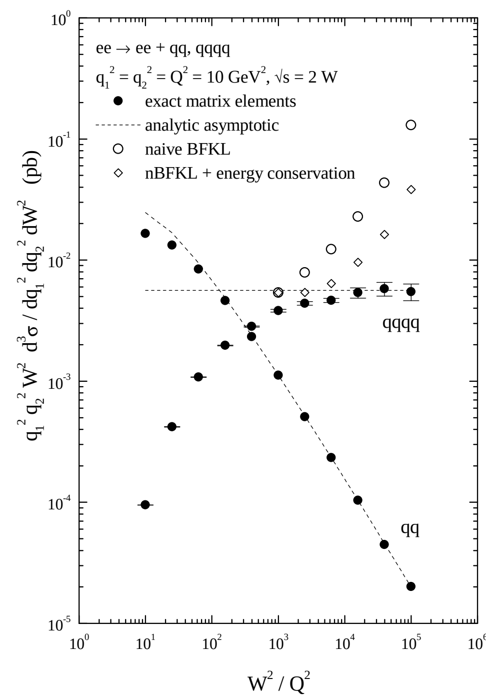

Figure 1. Exact (closed data points) and analytic asymptotic (dashed line) $e^+e^- \rightarrow e^+e^-q\bar{q}$ and $e^+e^- \rightarrow e^+e^-q\bar{q}q\bar{q}$ cross sections versus W^2/Q^2 at fixed $W^2/s = 1/4$. Also shown: analytic BFKL without (open circles) and with (open diamonds) energy conservation imposed.

events) with hadrons in between. With total center-of-mass energy s, photon virtuality $-Q^2$, and photon-photon invariant mass (= invariant mass of the final hadronic system) W^2, BFKL effects are expected in the kinematic regime where W^2 is large and

$$s >> Q^2 >> \Lambda^2_{QCD}.$$

At fixed order in QCD, the dominant process is four-quark production with t-channel gluon exchange (each photon couples to a quark box; the quark boxes are connected via the gluon). The corresponding BFKL contribution arises from diagrams with a gluon ladder attached to the t-channel gluon.

The relative contributions of fixed-order

QCD and BFKL are most easily understood by looking at

$$W^2 Q_1^2 Q_2^2 \, \frac{d^3\sigma}{dW^2 dQ_1^2 dQ_2^2}$$

as a function of W^2/Q^2 for fixed $\sqrt{s}/W$. The asymptotic regime then corresponds to large W^2/Q^2. This quantity is shown in Figure 1 for $Q_1^2 = Q_2^2 = Q^2 = 10\ \mathrm{GeV}^2$ and $\sqrt{s} = 2W$. The solid points are the QCD calculations of two-quark ('qq') and four-quark production ('qqqq'); we see that the latter dominates for large W^2/Q^2 and approaches a constant asymptotic value. In contrast, the analytic BFKL result, shown with open circles, rises well above that of fixed-order QCD. The diamonds show analytic BFKL with energy conservation imposed, but not exact kinematics; it can be interpreted as an upper limit for the Monte Carlo prediction, which is in progress.

It is important to note in Figure 1 that although BFKL makes a definite leading-order prediction for the behavior of the cross section as a function of W^2/Q^2, the origin in W^2/Q^2 (i.e., where BFKL meets asymptotic QCD) is *not* determined in leading order. We have chosen $W^2/Q^2 = 10^3\ \mathrm{GeV}^2$ as a reasonable value where the QCD behavior is sufficiently asymptotic for BFKL to become relevant, but another choice might be just as reasonable. Only when higher order corrections are computed can the BFKL prediction be considered unique.

From an experimental point of view, the cross section at fixed $\sqrt{s}$ is more directly relevant. Figure 2 shows $W^2 Q_1^2 Q_2^2 \, \frac{d^3\sigma}{dW^2 dQ_1^2 dQ_2^2}$ for a linear collider energy $\sqrt{s} = 500$ GeV. The solid lines show the exact fixed-order QCD prediction. The dashed line is the asymptotic four-quark production cross section, and the dotted line is the analytic BFKL prediction. Now we see that all of the curves fall off at large W, but the BFKL cross section lies well above the others.

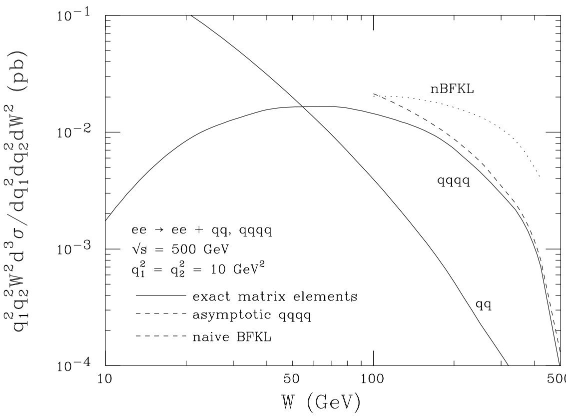

Figure 2. Exact (solid lines) and analytic asymptotic (dashed line) $e^+e^- \to e^+e^- q\bar{q}$ and $e^+e^- \to e^+e^- q\bar{q}q\bar{q}$ cross sections versus W^2/Q^2 at fixed $\sqrt{s} =$ 500 GeV. Also shown: analytic BFKL (dotted line).

3.1 $\gamma^*\gamma^*$ Scattering at LEP

The L3 collaboration at LEP have measured the $\gamma^*\gamma^*$ cross section by dividing the double-tagged e^+e^- cross section by the $\gamma^*\gamma^*$ luminosity.[5] They present their results, for $\sqrt{s} = 183$ and 189–202 GeV, with $Q^2 = 14\ \mathrm{GeV}^2$ and $Q^2 = 15\ \mathrm{GeV}^2$, respectively, as a function of $y = \ln(W^2/Q^2)$. In this variable the asymptotic QCD four-quark cross section is flat, and the analytic BFKL cross section rises, similarly to Figure 1. The data lie between the two predictions, and the higher statistics data at the higher energies show a clear rise in the data, though not as steep as predicted by analytic BFKL.

We expect that the BFKL Monte Carlo prediction (in progress) will be closer to the data. But one can also ask whether the asymptotic QCD limit for four-quark production is appropriate here. We compare the exact and asymptotic QCD curves at the LEP energy $\sqrt{s} = 183$ GeV in Figure 3 (note that this is the undivided e^+e^- cross section that includes the photon luminosity). The values of W corresponding to the LEP measurements range between about 15 and 90

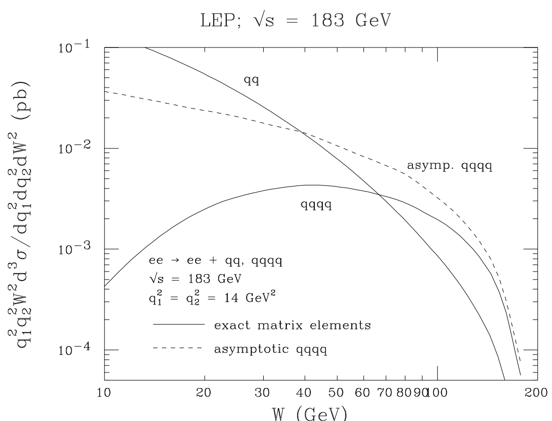

Figure 3. Exact (solid lines) and analytic asymptotic (dashed line) $e^+e^- \rightarrow e^+e^-q\bar{q}$ and $e^+e^- \rightarrow e^+e^-q\bar{q}q\bar{q}$ cross sections versus W^2/Q^2 at fixed $\sqrt{s} =$ 183 GeV.

GeV. Comparing four-quark predictions, we see that the exact curve is not close enough to the asymptotic in this region for the asymptotic QCD limit to be appropriate. Furthermore, the ratio of exact to asymptotic results — which is proportional to the $\gamma^*\gamma^*$ cross section — *rises* in this region. The QCD prediction is not flat at all.[c] Until the fixed-order QCD and BFKL Monte Carlo predictions are sorted out, it is not clear what we can conclude from the data.

4 Status of NLL Corrections

It is apparent that, although it is not yet clear whether BFKL is necessary to describe the data in hand, leading-order analytic BFKL is not sufficient. We need BFKL at next-to-leading order (strictly speaking, next-to-leading log order). This has been accomplished after 10 years of heroic efforts by Fadin, Lipatov and many others (see [10] for a review, complete references, and more details about what follows). The bad news is that the solutions appear to be large, unstable, and capable of giving negative cross sections.

[c]This does not automatically imply that fixed-order QCD describes the data, because there are unresolved normalization issues involved.

The good news is that there is much progress in understanding these problems, which can mostly be traced to the fact that at NLL gluons can be close together in rapidity, leading to collinear divergences. Several methods for solving this problem are summarized in [10], and the NLL BFKL corrections appear to be coming under control.

5 Conclusions

In summary, BFKL physics is a complicated business. Tests are being performed in a variety of present experiments (Tevatron, HERA, LEP) and there is potential for the future as well (LHC, LC). Unfortunately, comparisons between theory and experiment are not straightforward; leading order BFKL is insufficient, and subleading corrections such as kinematic constraints can be very important. A worst-case scenario which is not ruled out may be that we cannot reach sufficiently asymptotic regions in experiments to see unambiguous BFKL effects. However, reports of the demise of BFKL physics due to instability of the next-to-leading-order corrections are greatly exaggerated, and the source of the large corrections is understood and they are being brought under control. In summary, the jury on BFKL physics is still out, but there continues to be much progress.

6 Acknowledgments

Work supported in part by the U.S. Department of Energy and the U.S. National Science Foundation, under grants DE-FG02-91ER40685 and PHY-9600155.

References

1. L.N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 338; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP **45** (1977) 199; Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.

2. A.H. Mueller and H. Navelet, Nucl. Phys. **B282**, 727 (1987).
3. S.J. Brodsky, F. Hautmann and D.E. Soper, Phys. Rev. **D56**, 6957 (1997).
4. S. Abachi *et al.* [D0 Collaboration], Phys. Rev. Lett. **77**, 595 (1996).
5. M. Wadhwa for the L3 collaboration, these proceedings.
6. B. Pope for the D0 collaboration, these proceedings.
7. L.H. Orr and W.J. Stirling, Phys. Rev. **D56** (1997) 5875.
8. C.R. Schmidt, Phys. Rev. Lett. **78** (1997) 4531.
9. L.H. Orr and W.J. Stirling, Phys. Lett. **B429** (1998) 135; Phys. Lett. **B436** (1998) 372.
10. G. P. Salam, Acta Phys. Polon. **B30**, 3679 (1999); hep-ph/0005304, and references therein.